Internet Appendix for Semi-strong Factors in Asset Returns

Gregory Connor* Robert A. Korajczyk †

July 1, 2022

Abstract

This is the internet appendix for "Semi-strong Factors in Asset Returns" - *Journal of Financial Econometrics.*

*School of Business, Maynooth University, Maynooth, Ireland. E-mail: gregory.connor@mu.ie.

†Kellogg School of Management, Northwestern University, 2211 Campus Drive, Evanston, IL, 60208 USA. Tel: 1-847-491-8336: E-mail: r-korajczyk@kellogg.northwestern.edu.

1 A.1: Derivation of the estimation variance of bias-adjusted squared estimated beta

We now derive the expression for the estimation variance of the adjusted squared beta estimator for a given i, that is:

$$Var[(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}] = E[(B_{ij} + \sum_{t=1}^{T} m_{jt}\varepsilon_{it} - \sum_{t=1}^{T}\sum_{\tau=1}^{T} p_{t\tau}\varepsilon_{it}\varepsilon_{i\tau})^2] \tag{A1}$$

in the special case where the idiosyncratic returns are cross-sectionally independent. Expanding out the left hand side of (A1) into its components:

$$Var[(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}] = Var[(\widehat{B}_{ij})^2] + (\overline{m}_{2j})^2 Var[\widehat{\sigma}^2_{\varepsilon_i}] - 2\overline{m}_{2j}Cov[(\widehat{B}_{ij})^2, \widehat{\sigma}^2_{\varepsilon_i}]. \tag{A2}$$

We compute each of the three components on the right-hand side of (A2) separately, beginning with the first. Note that:

$$Var[(\widehat{B}_{ij})^2] = E[((\widehat{B}_{ij})^2 - \overline{m}_{2j}\sigma^2_{\varepsilon_i} - B^2_{ij})^2]. \tag{A3}$$

Expressing $(\widehat{B}_{ij})^2$ using (A1) :

$$(\widehat{B}_{ij})^2 = B^2_{ij} + 2B_{ij}\sum_{t=1}^{T} m_{jt}\varepsilon_{it} + \sum_{t=1}^{T}\sum_{\tau=1}^{T} m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau}. \tag{A4}$$

Inserting (A4) into the right-hand side of (A3) gives:

$$E[((\widehat{B}_{ij})^2 - \overline{m}_{2j}\sigma^2_{\varepsilon_i} - B^2_{ij})^2] = E[(2B_{ij}\sum_{t=1}^{T} m_{jt}\varepsilon_{it} + \sum_{t=1}^{T}\sum_{\tau=1}^{T} m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau} - \overline{m}_{2j}\sigma^2_{\varepsilon_i})^2]. \tag{A5}$$

Expanding out the square on the right-hand side of (A5) gives three square terms and three cross terms:

$$
\begin{aligned}
& E[(2B_{ij}\sum_{t=1}^{T} m_{jt}\varepsilon_{it} + \sum_{t=1}^{T}\sum_{\tau=1}^{T} m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau} - \overline{m}_{2j}\sigma^2_{\varepsilon_i})^2] \\
= \quad & 4B_{ij}^2\sum_{t=1}^{T}\sum_{\tau=1}^{T} m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau} + & \text{(A6)} \\
& \sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T}\sum_{u=1}^{T} m_{jt}m_{j\tau}m_{js}m_{ju}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is}\varepsilon_{iu} + & \text{(A7)} \\
& (\overline{m}_{2j}\sigma^2_{\varepsilon_i})^2 + & \text{(A8)} \\
& 4B_{ij}\sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T} m_{jt}m_{j\tau}m_{js}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is} - & \text{(A9)} \\
& 4B_{ij}\overline{m}_{2j}\sigma^2_{\varepsilon_i}\sum_{t=1}^{T} m_{jt}\varepsilon_{it} - & \text{(A10)} \\
& 2\overline{m}_2\sigma^2_{\varepsilon_i}\sum_{t=1}^{T}\sum_{\tau=1}^{T} m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau}. & \text{(A11)}
\end{aligned}
$$

We will consider each of the six additive components above in order. For (A6), all the cross -terms $E[\varepsilon_{it}\varepsilon_{i\tau}]$, $t \neq \tau$ have expectation of zero and all the pure terms $E[\varepsilon^2_{it}]$ have expectation $\sigma^2_{\varepsilon_i}$, giving

$$E[4B_{ij}^2\sum_{t=1}^{T}\sum_{\tau=1}^{T} m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau}] = 4B_{ij}^2\overline{m}_{2j}\sigma^2_{\varepsilon_i}.$$

For (A7), there is one pure term where all four time indices are equal and three cross-product terms where two pairs of time indices are equal:

$$E[\sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T}\sum_{u=1}^{T} m_{jt}m_{j\tau}m_{js}m_{ju}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is}\varepsilon_{iu}] = \overline{m}_{4j}E[\varepsilon^4_i] + 3\overline{m}_{22j}(\sigma^2_{\varepsilon_i})^2.$$

(A8) is in a simple form already. For (A9), the only nonzero expectation is the pure sum when all three time indices are equal:

$$E[4B_{ij}\sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T} m_{jt}m_{j\tau}m_{js}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is}] = 4B_{ij}\overline{m}_{3j}E[\varepsilon^3_i].$$

Term (A10) has expectation zero. For (A11):

$$E[2\overline{m}_{2j}\sigma^2_{\varepsilon_i}\sum_{t=1}^{T}\sum_{\tau=1}^{T}m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau}] = 2(\overline{m}_{2j}\sigma^2_{\varepsilon_i})^2.$$

Adding together the components:

$$\begin{aligned} Var[(\widehat{B}_{ij})^2] &= 4B^2_{ij}\overline{m}_{2j}\sigma^2_{\varepsilon_i} + \overline{m}_{4j}E[\varepsilon^4_i] + 3\overline{m}_{22j}(\sigma^2_{\varepsilon_i})^2 + (\overline{m}_{2j}\sigma^2_{\varepsilon_i})^2 + \\ &\quad 4B_{ij}\overline{m}_{3j}E[\varepsilon^3_i] - 2(\overline{m}_{2j}\sigma^2_{\varepsilon_i})^2 \\ &= 4\overline{m}_{2j}B^2_{ij}\sigma^2_{\varepsilon_i} + 4B_{ij}\overline{m}_{3j}E[\varepsilon^3_i] + \overline{m}_{4j}E[\varepsilon^4_i] + (3\overline{m}_{22j} - (\overline{m}_{2j})^2)(\sigma^2_{\varepsilon_i})^2. \quad \text{(A12)} \end{aligned}$$

Next we derive the second term of (A2). Recall that $\widehat{\sigma}^2_{\varepsilon_i} = \sum\limits_{t=1}^{T}\sum\limits_{\tau=1}^{T}p_{t\tau}\varepsilon_{it}\varepsilon_{i\tau}$. Taking the expectation of the square minus the squared expectation:

$$Var[\widehat{\sigma}^2_{\varepsilon_i}] = E[\sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T}\sum_{u=1}^{T}p_{t\tau}p_{su}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is}\varepsilon_{iu}] - (\sigma^2_{\varepsilon_i})^2,$$

and using that ε_{it} is independent across time with zero mean:

$$\begin{aligned} &= E[\varepsilon^4_i]\sum_{t=1}^{T}p^2_{tt} + (\sigma^2_{\varepsilon_i})^2(\sum_{t=1}^{T}\sum_{\tau\neq t}2p^2_{t\tau} + p_{tt}p_{\tau\tau}) - (\sigma^2_{\varepsilon_i})^2 \\ &= \overline{p}_2E[\varepsilon^4_i] + ((p\times p) - 1)(\sigma^2_{\varepsilon_i})^2 \qquad \text{(A13)} \end{aligned}$$

The last term in (A2) involves the covariance between $(\widehat{B}_{ij})^2$ and $\widehat{\sigma}^2_{\varepsilon_i}$. Taking the product of their de-meaned values:

$$Cov[(\widehat{B}_{ij})^2, \widehat{\sigma}^2_{\varepsilon_i}] = E[(\sum_{t=1}^{T}\sum_{\tau=1}^{T}p_{t\tau}\varepsilon_{it}\varepsilon_{i\tau} - \sigma^2_{\varepsilon_i})(2B_{ij}\sum_{t=1}^{T}m_{jt}\varepsilon_{it} + \sum_{t=1}^{T}\sum_{\tau=1}^{T}m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau})]. \qquad \text{(A14)}$$

Expanding out the four terms in (A14):

$$\begin{aligned} = \quad & 2B_{ij}E[\sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T}p_{t\tau}m_{js}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is}] + \\ & E[\sum_{t=1}^{T}\sum_{\tau=1}^{T}\sum_{s=1}^{T}\sum_{u=1}^{T}p_{t\tau}m_{js}m_{ju}\varepsilon_{it}\varepsilon_{i\tau}\varepsilon_{is}\varepsilon_{iu}] \\ & -\sigma^2_{\varepsilon_i}2B_{ij}E[\sum_{t=1}^{T}m_{jt}\varepsilon_{it}] - \sigma^2_{\varepsilon_i}E[\sum_{t=1}^{T}\sum_{\tau=1}^{T}m_{jt}m_{j\tau}\varepsilon_{it}\varepsilon_{i\tau}]. \end{aligned}$$

and then using the ε_{it} is independent through time with zero mean:

$$\begin{aligned} = \quad & 2B_{ij}E[\varepsilon_i^3]\sum_{t=1}^{T}p_{tt}m_{jt} + E[\varepsilon_i^4]\sum_{t=1}^{T}p_{tt}m_{jt}^2 \\ & +(\sigma^2_{\varepsilon_i})^2\sum_{t=1}^{T}\sum_{\tau\neq t}^{T}(2p_{t\tau}m_{jt}m_{j\tau} + p_{tt}m_{j\tau}^2) - (\sigma^2_{\varepsilon_i})^2\sum_{t=1}^{T}m_{jt}^2 \\ = \quad & 2B_{ij}\overline{pm}_jE[\varepsilon_i^3] + \overline{pm}_{2j}E[\varepsilon_i^4] + ((p\times m) - \overline{m}_{2j})(\sigma^2_{\varepsilon_i})^2. \end{aligned} \tag{A15}$$

Now we collect the terms from (A12), (A13), and (A15) and combine them into (A1), multiplying (A13) terms by $(\overline{m}_{2j})^2$ and (A15) terms by $-2\overline{m}_{2j}$, giving:

$$\begin{aligned} Var[(\widehat{B}_{ij})^2 - \overline{m}_{j2}\widehat{\sigma}^2_{\varepsilon_i}] \quad = & \\ & 4\overline{m}_{2j}B_{ij}^2\sigma^2_{\varepsilon_i} + 4B_{ij}\overline{m}_{3j}E[\varepsilon_i^3] + \overline{m}_{4j}E[\varepsilon_i^4] + (3\overline{m}_{22j} - (\overline{m}_{2j})^2)(\sigma^2_{\varepsilon_i})^2 \\ & +(\overline{m}_{2j})^2(\overline{p}_2E[\varepsilon_i^4] + ((p\times p) - 1)(\sigma^2_{\varepsilon_i})^2) \\ & -2\overline{m}_{2j}(2B_{ij}\overline{pm}_jE[\varepsilon_i^3] + \overline{pm}_{2j}E[\varepsilon_i^4] + ((p\times m) - \overline{m}_{2j})(\sigma^2_{\varepsilon_i})^2)). \end{aligned}$$

Sorting by the moments and simplifying, this becomes:

$$\begin{aligned}
Var[(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}] &= c_{1ij}\sigma^2_{\varepsilon_i} + c_{2ij}E[\varepsilon_i^3] + c_{3j}E[\varepsilon_i^4] + c_{4j}(\sigma^2_{\varepsilon_i})^2 \\
c_{1ij} &= 4\overline{m}_{2j}B^2_{ij} \\
c_{2ij} &= 4B_{ij}\overline{m}_{3j} - 4\overline{m}_{2j}B_{ij}(\overline{pm}_j) \\
c_{3j} &= \overline{m}_{4j} + (\overline{m}_{2j})^2\overline{p}_2 - 2\overline{m}_{2j}(\overline{pm}_{2j}) \\
c_{4j} &= 3\overline{m}_{22j} + (\overline{m}_{2j})^2(p \times p) - 2\overline{m}_{2j}(p \times m).
\end{aligned}$$

2 A.2: Application of the Central Limit Theorem to the bias-adjusted mean-squared beta statistic

In this section of the appendix we show that the assumptions made in the text justify the central limit theorem for the bias-adjusted mean-squared beta statistic, $\widehat{\chi}_j$, which we call herein the chi statistic. We use the version of the central limit statistic due to White and Domowitz; see also White (1984).

The chi statistic is an average of error-adjusted squared estimated betas (equation (15) in the paper). In order for the central limit theorem to apply to this average (for a given $j = 1, k$) we need to confirm three conditions (see White (1984, p. 124)):

Condition 1: Let $\alpha(m)$ denote the strong mixing process for (equation (15) in the paper), then $\alpha(m)$ must be of size $r/(r-1)$ for some $r > 1$.

Condition 2: $E[|(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}|^{2r}] < \Delta$ for some fixed finite Δ for all i using the r from Condition 1.

Condition 3: Assumption 6 (n times the estimation variance converges to a constant) must hold.

For Condition 1, note that we have assumed (by Assumption 4) that the mixing process for ε_i obeys $\alpha(m) = 0$ for all $m \geq m^*$. The ith term in the sequence of random variables (equation (15) in the paper) is a nonstochastic function of the $T-$vector of realizations $\{\varepsilon_{it}\}_{t=1,T}$ and therefore inherits the property $\alpha^*(m) = 0$ for all $m \geq m^*$ since nonstochastic functions of independent random variables are independent. This implies that (equation (15) in the paper) is a strong mixing process of size $r/(r-1)$ for any $r > 1$. We choose $r = 1 + \frac{\delta}{2}$ where δ comes from Assumption 5.

For Condition 2, consider the $2r$th absolute moment of (equation (15) in the paper) with $r = 1 + \frac{\delta}{2}$:

$$E[|B_{ij}^2 + (\sum_{t=1}^{T} 2B_{ij}m_{jt}\varepsilon_{it} + \sum_{\tau=1}^{T}(m_{jt}m_{j\tau} - p_{t\tau})\varepsilon_{it}\varepsilon_{i\tau})|^{2+\delta}]. \tag{A16}$$

Note that (A16) is a constant (over i) polynomial function of B_{ij} and the $T-$vector $\{\varepsilon_{it}\}_{t=1,T}$. It is easy to see that this expression has bounded absolute moments, for all finite-order moments. For simplicity we can use Δ in Condition 2 derived from its maximum possible value within the compact support of B_{ij} and $\{\varepsilon_{it}\}_{t=1,T}$:

$$\Delta > \underset{B_{ij},\{\varepsilon_{it}\}_{t=1,T}}{\arg\max} |B_{ij}^2 + (\sum_{t=1}^{T} 2B_{ij}m_t\varepsilon_{it} + \sum_{\tau=1}^{T}(m_t m_\tau - p_{t\tau})\varepsilon_{it}\varepsilon_{i\tau})|^{2+\delta}.$$

which completes the sufficient conditions for the central limit theorem.

In our Assumption 6 we assume directly that n times the estimation variance of the chi statistic converges to a finite value. Now we consider the special case of independently distributed ε_i, implying a strict rather than approximate factor model, and give explicit conditions on cross-sectional average moments guaranteeing Assumption 6. We also derive the exact expression for ψ_j in this case.

Independence of ε_i implies independence of $(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}$ so that the variance of the cross-sectional average $\widehat{\chi}$ is $\frac{1}{n}$ times the average of the variances:

$$Var[\widehat{\chi}_j] = \frac{1}{n^2}\sum_{i=1}^{n} Var[(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}] \tag{A17}$$

Multiplying both sides of (A17) by n and using the expression for $Var[(\widehat{B}_{ij})^2 - \overline{m}_{2j}\widehat{\sigma}^2_{\varepsilon_i}$:

$$nVar[\widehat{\chi}_j] = \frac{1}{n}\sum_{i=1}^{n} c_{1ij}\sigma^2_{\varepsilon_i} + c_{2ij}E[\varepsilon_i^3] + c_{3j}E[\varepsilon_i^4] + c_{4j}(\sigma^2_{\varepsilon i})^2.$$

Using the expressions for $c_{1ij}, c_{2ij}, c_{3j}, c_{4j}$ from above, the convergence of $nVar[\widehat{\chi}_j]$ follows from the convergence of the following cross-sectionally averaged moments:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} B^2_{ij}\sigma^2_{\varepsilon_i}$$
$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} B_{ij}E[\varepsilon_i^3]$$
$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} E[\varepsilon_i^4]$$
$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} (\sigma^2_{\varepsilon i})^2$$

Given that these four limiting expressions converge to finite values, it follows that the asymptotic variance ψ_j also has a finite convergent value:

$$\psi_j = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} c_{1ij}\sigma^2_{\varepsilon_i} + c_{2ij}E[\varepsilon_i^3] + c_{3j}E[\varepsilon_i^4] + c_{4j}(\sigma^2_{\varepsilon i})^2.$$

A.3: Discussion of the Bootstrap Estimation of the Chi Statistic's Estimation Variance

This section of the appendix discusses the application of random-case bootstrapping to our model. We use the technique to estimate the variance of our chi-statistic ((equation (19) in the paper)).

We have derived an exact formula for the variance of the chi-statistic under a strict factor model assumption, but it is a function of the third and fourth moments of asset-specific returns, which are difficult to estimate consistently from time-series regression residuals, and also this formula does not apply to case of an approximate factor model. The boostrapping approach to estimating $var[\widehat{\chi}_j]$ is natural in this context. Bootstrapping uses the full available sample to estimate this variance directly from the sample data. To implement this estimator we act as if the time series i.i..d. multivariate probability distribution generating returns and factors consists of an $n+k$-vector process, s_τ which has T discrete possible realizations

$$s_\tau = [vec[R_\tau], vec[F_\tau]].$$

The discrete set of potential state realizations for s_τ are assumed equal to our observed sample values, and each of these T random states is assumed to have equal probability $\frac{1}{T}$. This is fully consistent with all of our factor model assumptions on returns.

Within the context of this discretized probability distribution we can derive the exact variance of the chi-statistic. We note that the chi-statistic is a function of a sample of T random realizations of the state process:

$$\widehat{\chi}_j = f(s_1, ..., s_T)$$

where $s_1, ..., s_T$ are independently and identically distributed realizations, and the formula for $f(\cdot)$ is the estimation formula for $\widehat{\chi}_j$. Using the discretized probability distribution, the exact variance of $\widehat{\chi}_j$ is:

$$var[\widehat{\chi}_j] = \frac{1}{T^T}\sum_{S^T} f(s_1, ..., s_T)^2 - (\frac{1}{T^T}\sum_{S^T} f(s_1, ..., s_T))^2 \tag{A18}$$

where the averages run across the set of all $T-$tuples $s_1, ..., s_T$. Computing (A18) is straightforward but impractical since with $T = 1200$ (roughly our sample size) there are 1200^{1200} terms in the averages. However the averages are easily and closely approximated by simply averaging over a large number of random draws of $(s_1, ..., s_T)$; we use $50,000$ draws in each subperiod.

Section A.4: Monte Carlo Simulation: Failure of standard factor estimation methods in recovering semi-strong factors

Figure A.1 plots the R^2 values for the regression of the estimated factors on the true factors (equation (11) in the paper). The blue curve is for the first estimated factor and the orange curve is for the second estimated factor. The Figure shows how the R^2 values change as we change $\alpha, \rho, T, n, \sigma_\varepsilon$ holding the other parameters at the base-case values used for Table 1 ($\alpha = 0.45, \rho = 0.0, T = 1250, n = 3000, \sigma_\varepsilon = 0.35$).

Section A.5: Monte Carlo simulation of the test statistic, equation (equation (21) in the paper)

Figure A.2 plots the empirical cumulative distribution of the test statistic $\widehat{\chi}^s$ in equation (21), across 50000 simulations, along with the standard normal CDF.

REFERENCES

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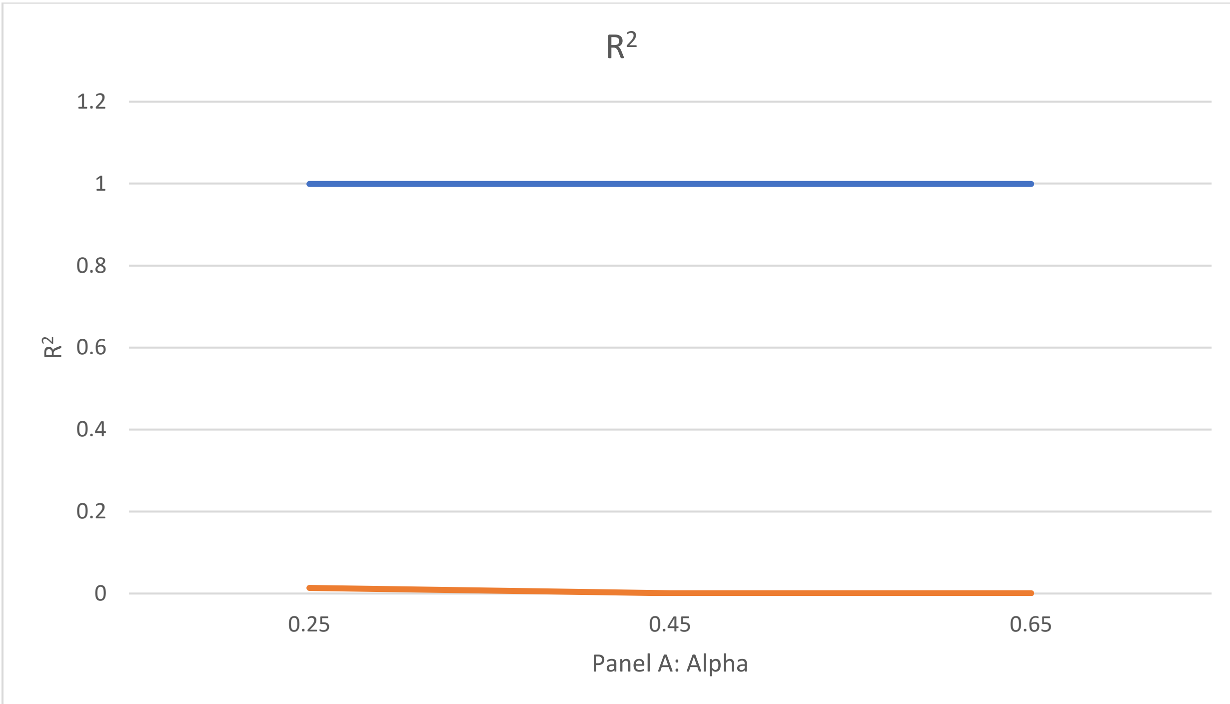
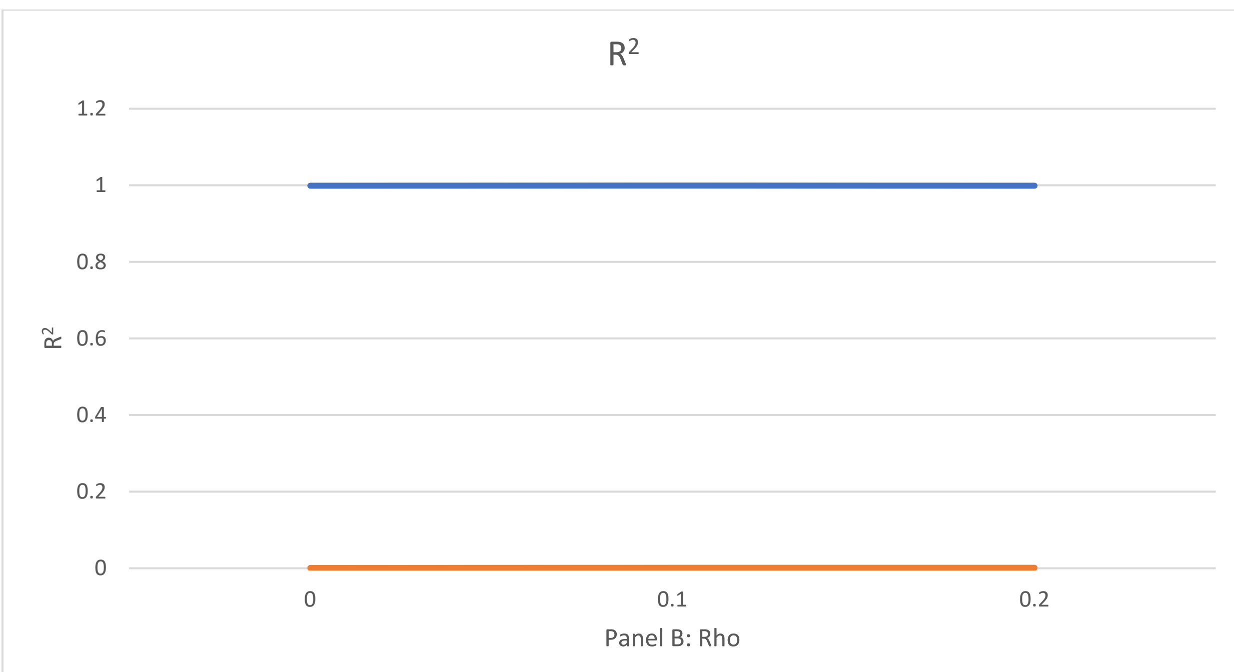
White, H., 1984. *Asymptotic Theory for Econometricians*, London: Academic Press.

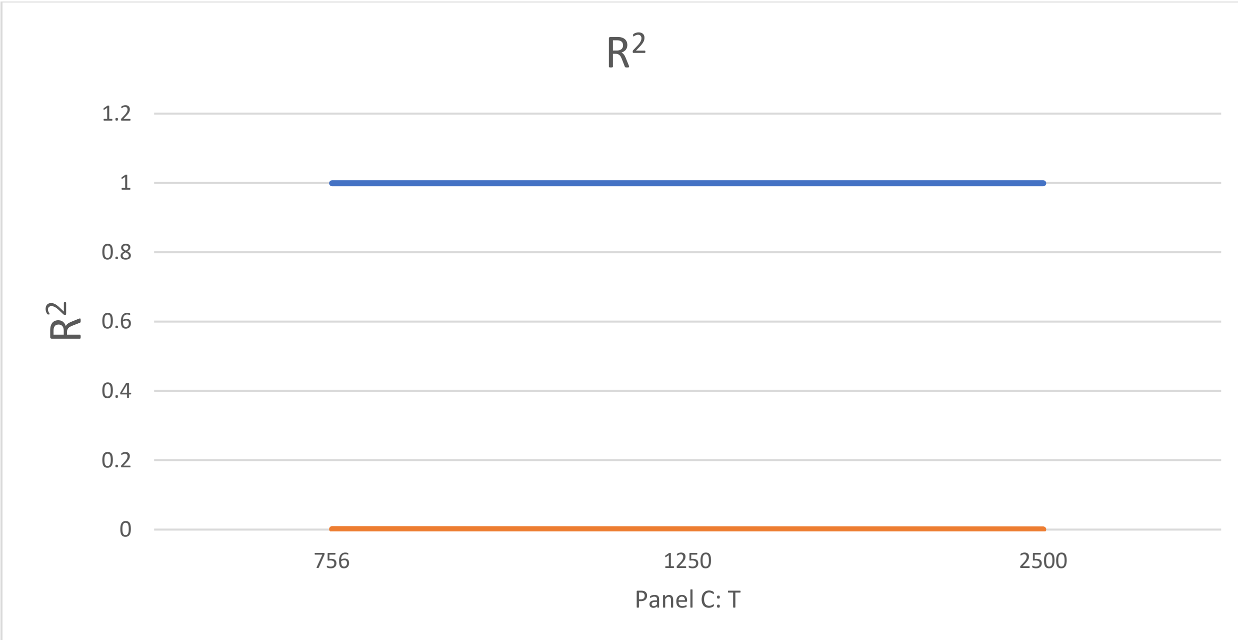
Figure A.1: R^2 of Estimated factors 1 (blue curve) and 2 (orange curve) on true factors. Each panel varies one parameter while holding the others at the base case: α=0.45, ρ=0.0, T=1250, n=3000, σ_ε=0.35.



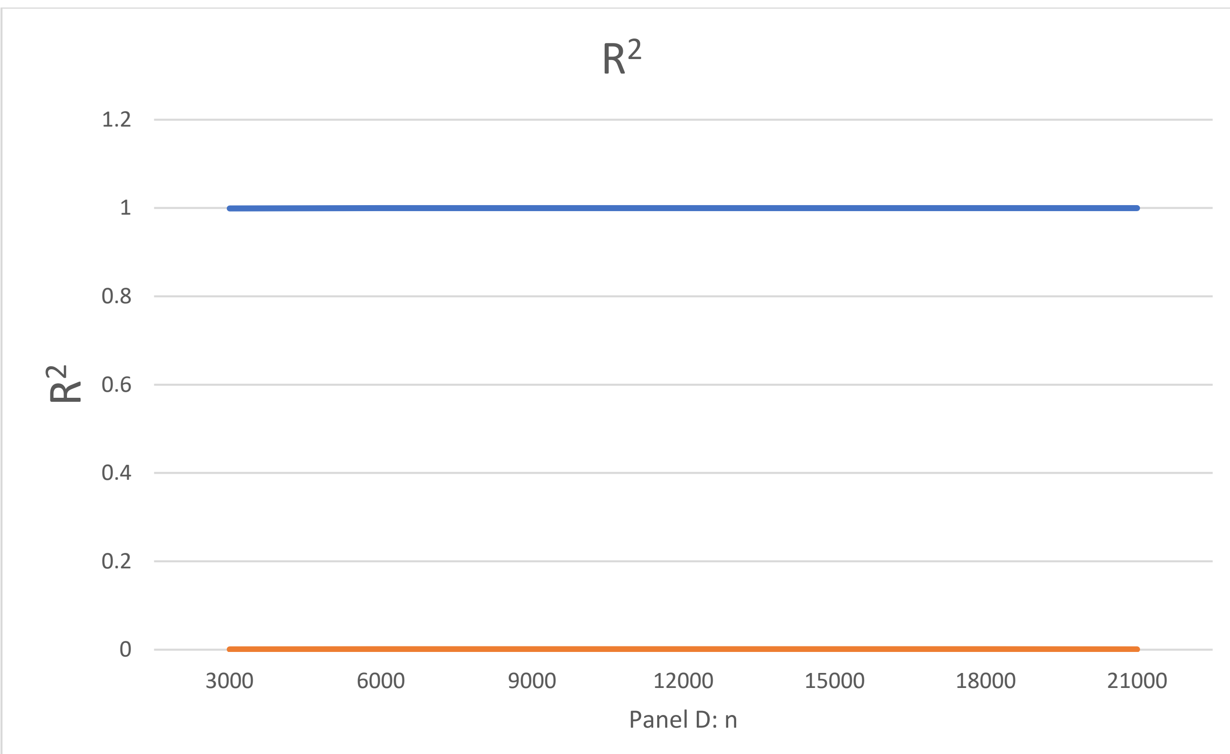
Panel A – R^2 of Estimated factors 1 (blue curve) and 2 (orange curve) on true factors. ρ=0.0, T=1250, n=3000, σ_ε=0.35.



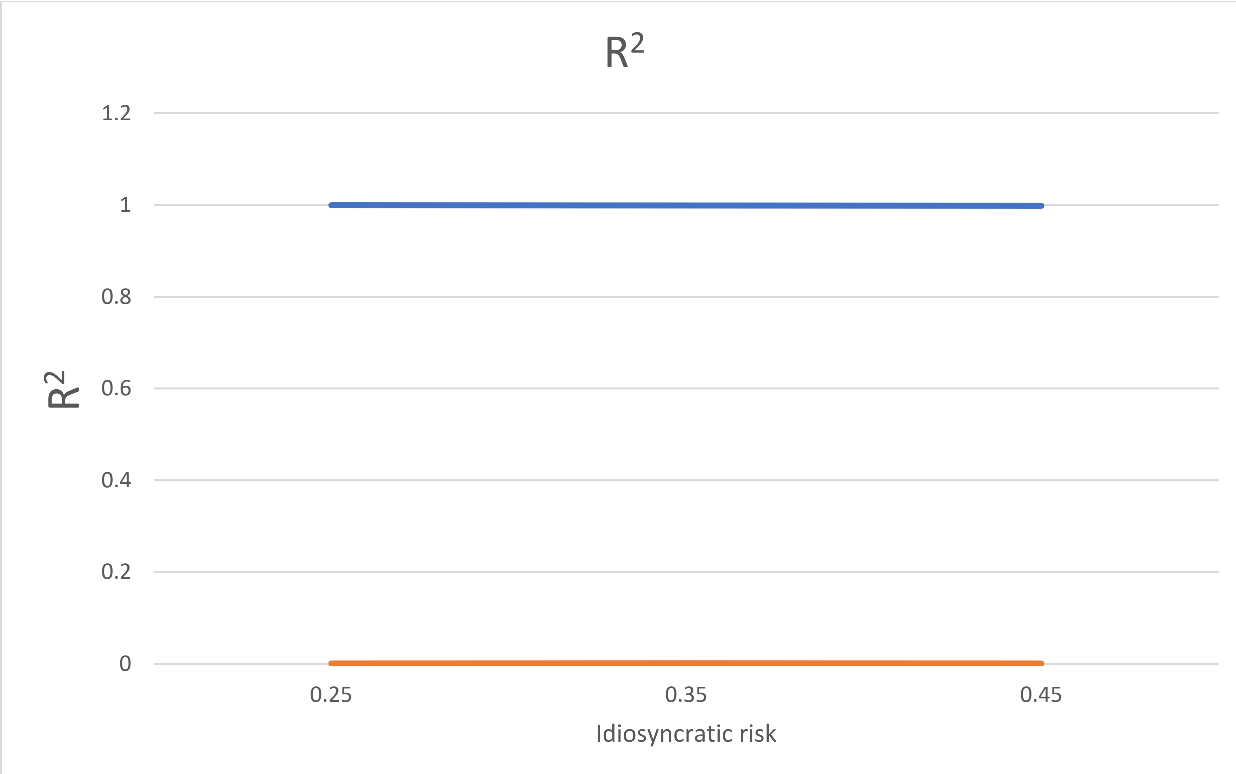
Panel B – R^2 of Estimated factors 1 (blue curve) and 2 (orange curve) on true factors. α=0.45, T=1250, n=3000, σ_ε=0.35.



Panel C – R^2 of Estimated factors 1 (blue curve) and 2 (orange curve) on true factors. α=0.45, ρ=0.0, n=3000, σ_ε=0.35.

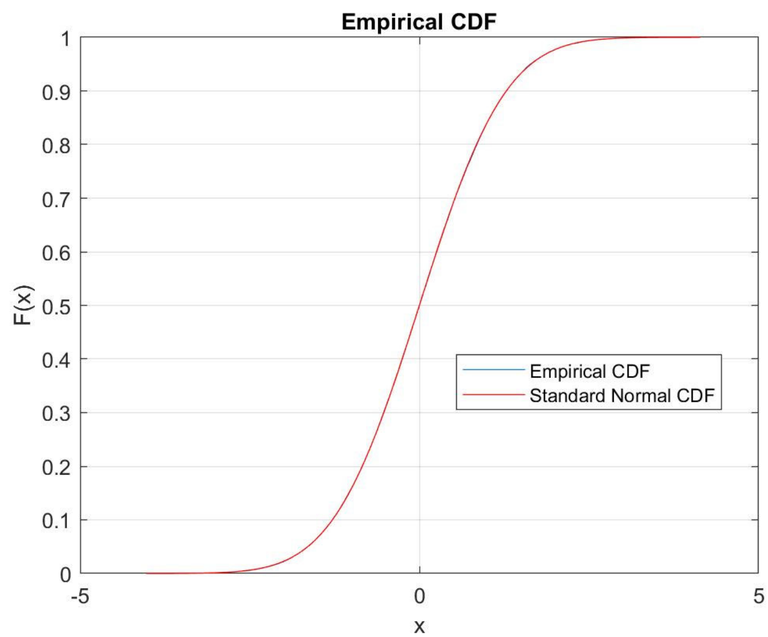


Panel D – R^2 of Estimated factors 1 (blue curve) and 2 (orange curve) on true factors. α=0.45, ρ=0.0, T=1250, σ_ε=0.35.

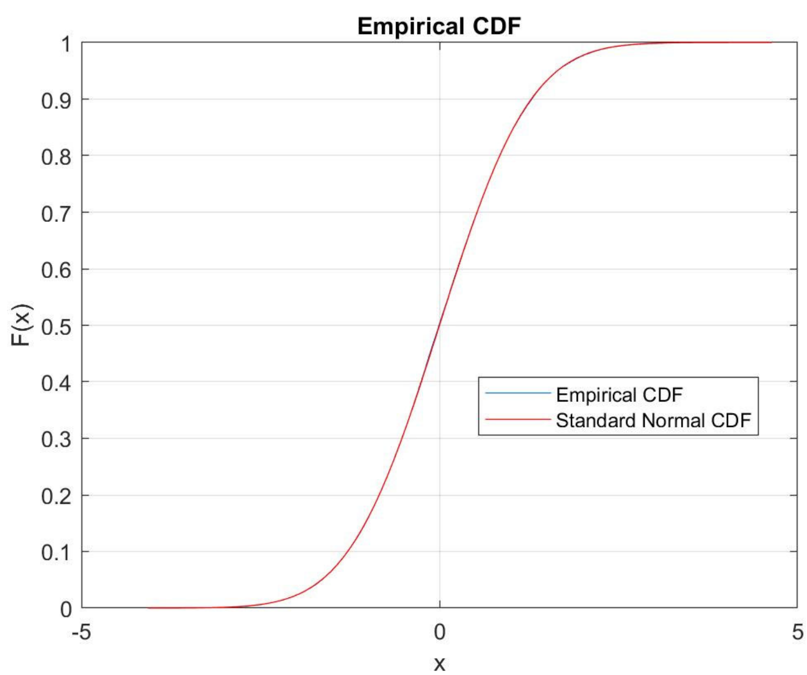


Panel E – R^2 of Estimated factors 1 (blue curve) and 2 (orange curve) on true factors. α=0.45, ρ=0.0, T=1250, n=3000..

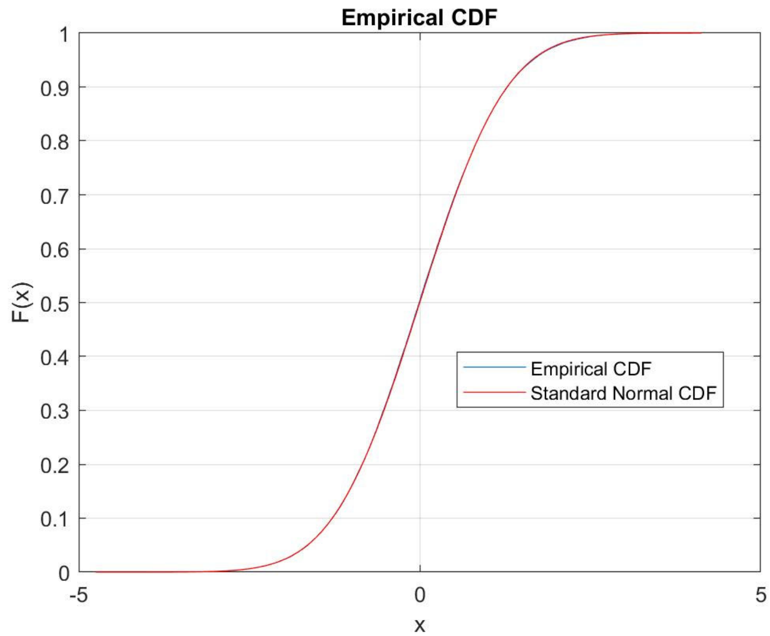
Figure A.2: Plot of standard normal distribution and the empirical distribution of standardized sum of squared betas, $\widehat{\chi}^{s}=\dfrac{\widehat{\chi}_j-\chi_j}{\psi_j^{0.5}}$, for factors j = 1, 2, …, 10 across 50,000 simulations.



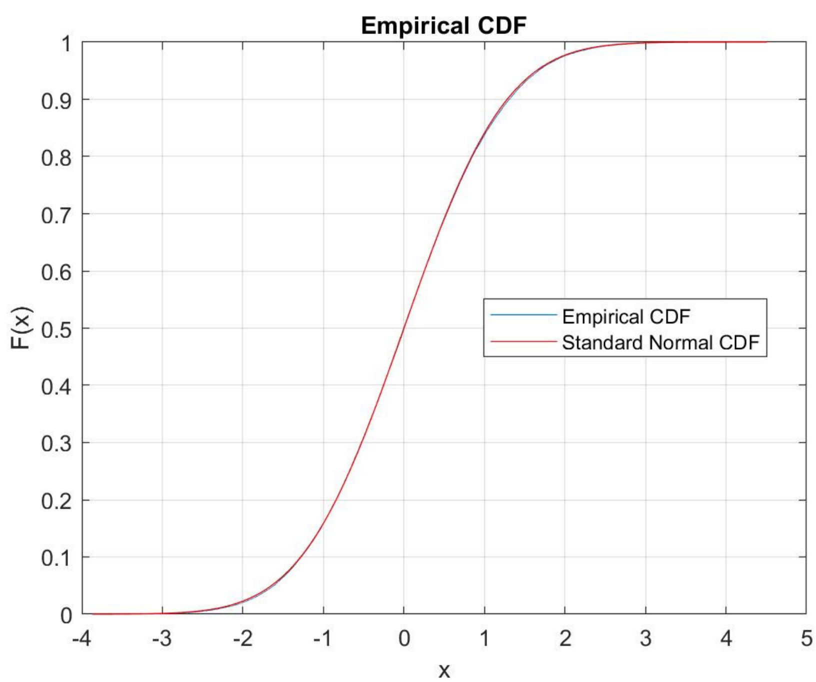
Panel A –*Market F*actor (blue curve) *vs. standard normal*.



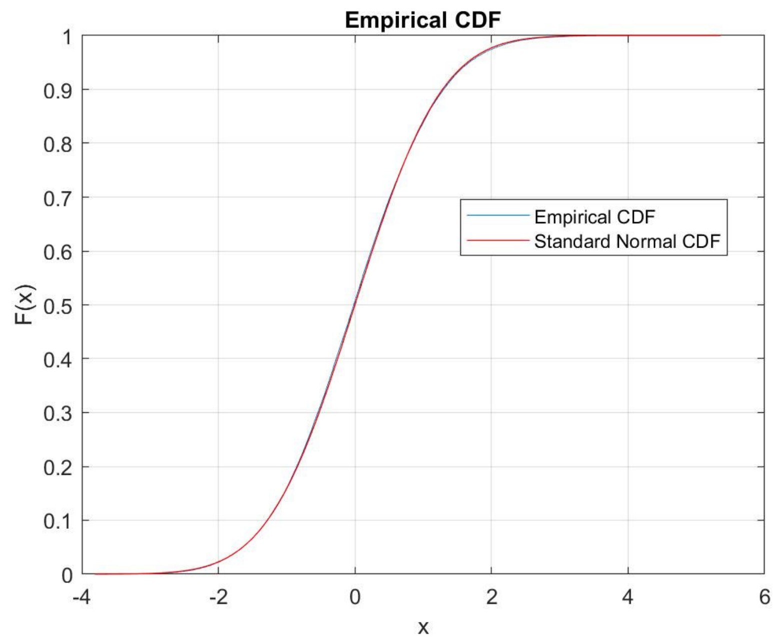
Panel B - SMB Factor (blue curve) vs. standard normal.



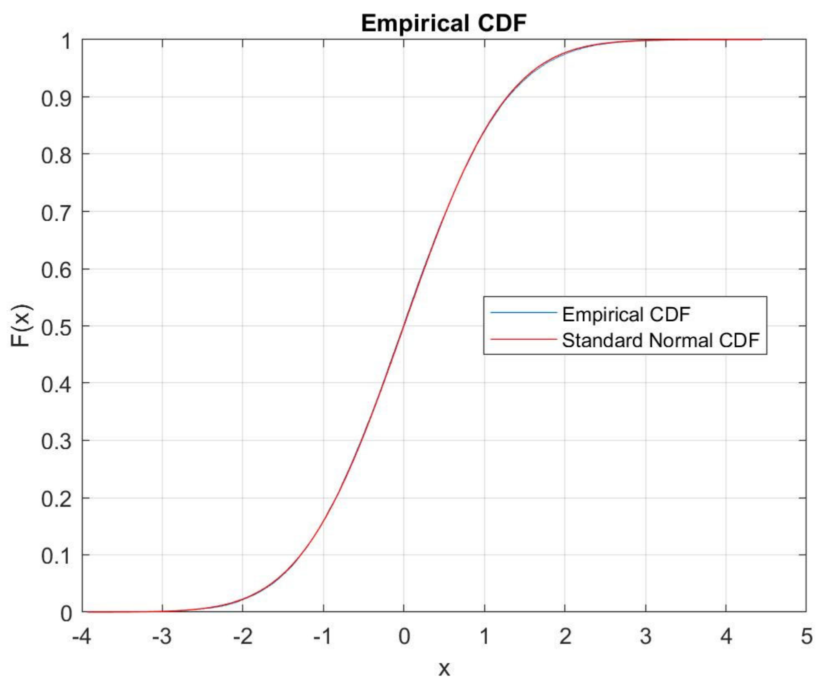
Panel C –*HML* Factor (blue curve) *vs. standard normal*.



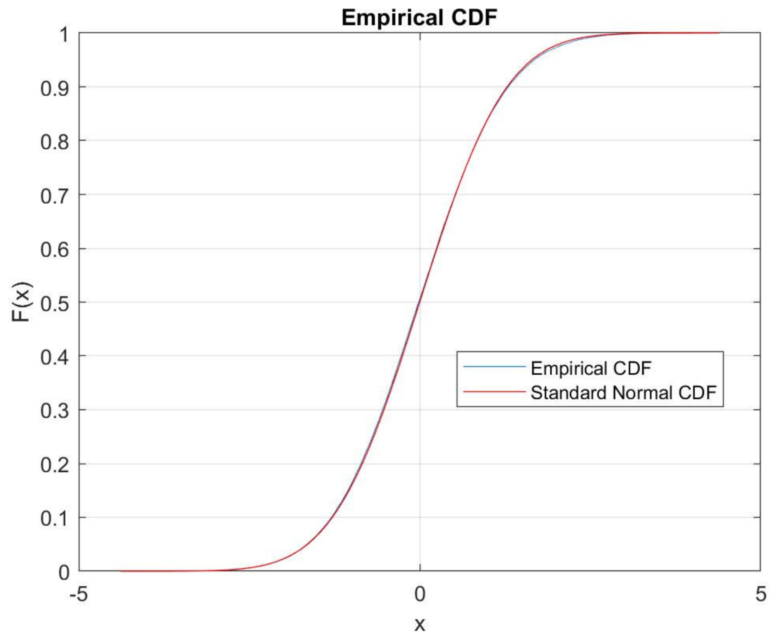
Panel D - USD Factor (blue curve) vs. standard normal.



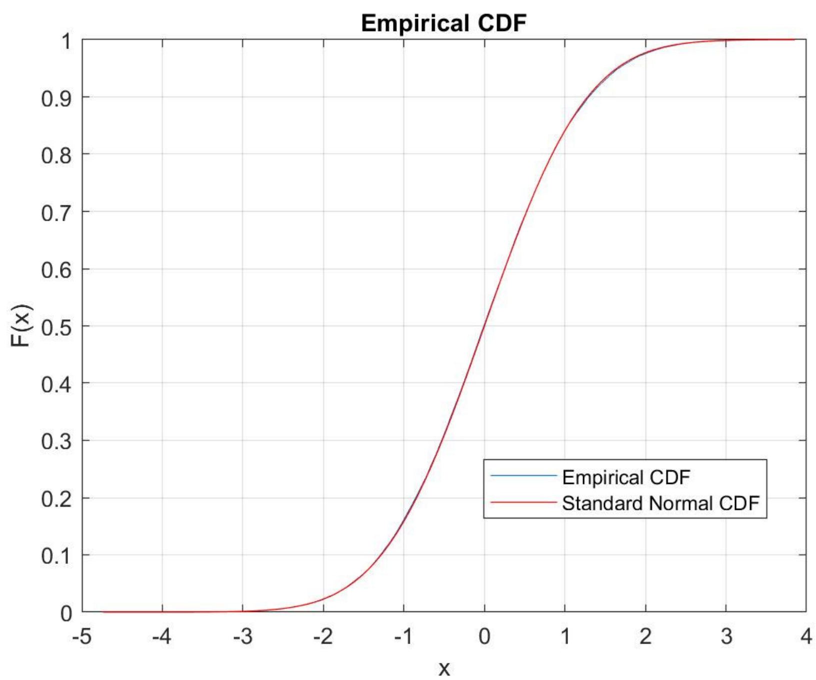
Panel E –*GBP* Factor (blue curve) *vs. standard normal*.



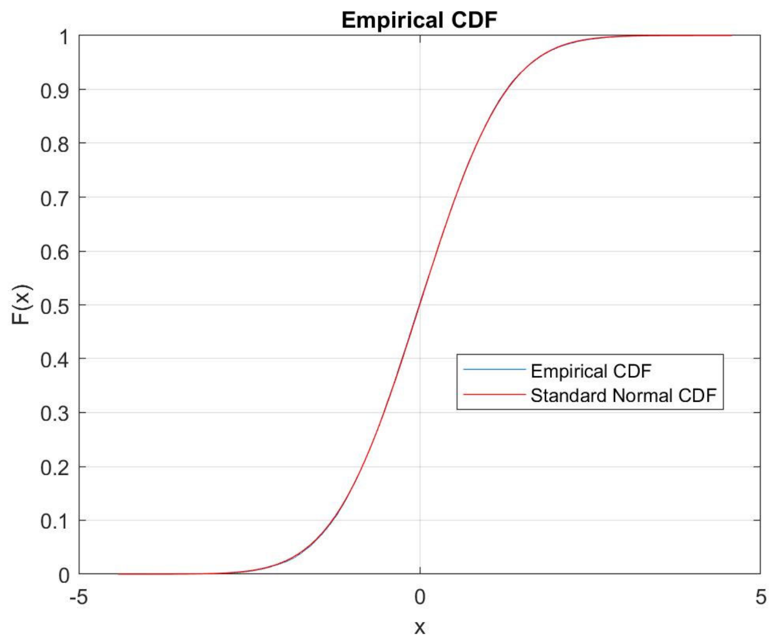
Panel F - JPY Factor (blue curve) vs. standard normal.



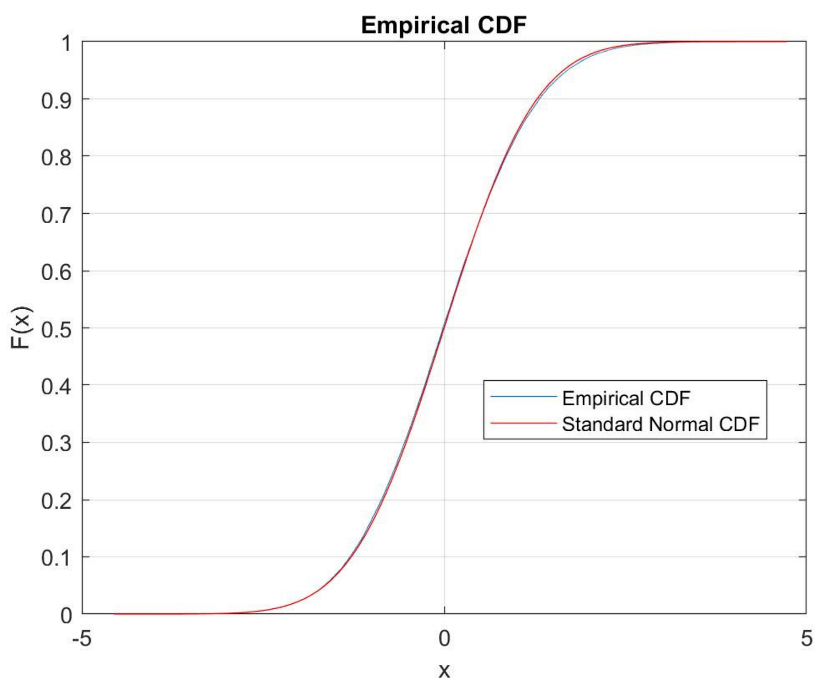
Panel G –*KRW* Factor (blue curve) *vs. standard normal*.



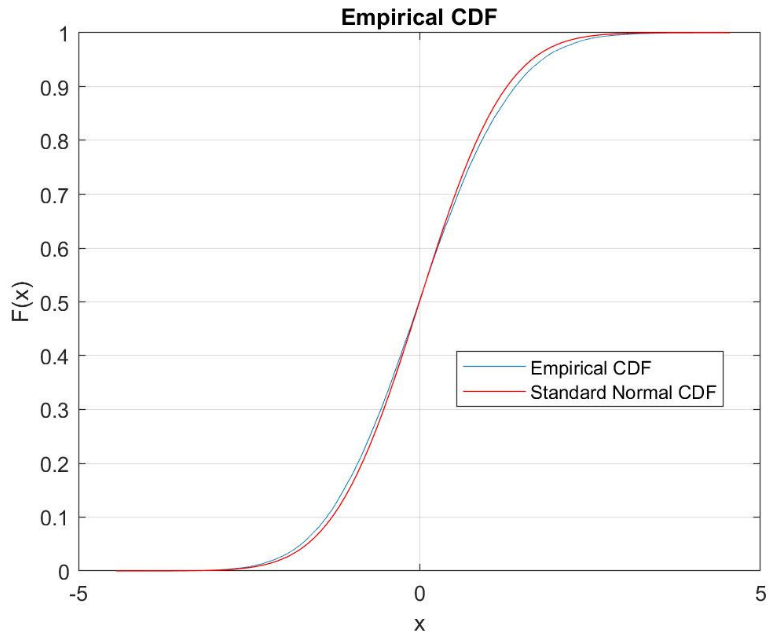
Panel H - Oil Factor (blue curve) vs. standard normal.



Panel I –*Gold* Factor (blue curve) *vs. standard normal*.



Panel J - Aluminum Factor (blue curve) vs. standard normal.



Panel K –Lumber Factor (blue curve) *vs. standard normal*.